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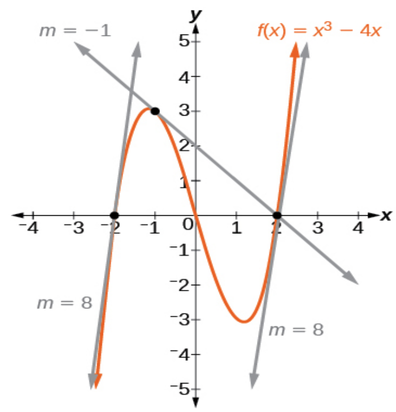

Day 1	Day 2	Day 3	Day 4	Day 5
Chapter 4 Lesson 5 Graphing Damped Trigonometric Functions	Chapter 4 Lesson 5 Graphing Damped Trigonometric Functions	Chapter 4 Lesson 7 The Law of Sines and the Law of Cosines with AAS, ASA, SAS, SSS	Chapter 4 Lesson 7 The Law of Sines and the Law of Cosines with SSA	Chapter 4 Lesson 7 The Law of Sines and the Law of Cosines with SSA
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Chapter 4 Lesson 7 The Law of Sines and the Law of Cosines with SSS, SAS to find Area of a Triangle	Chapter 4 Lesson 6 Graph inverse trigonometric functions	Chapter 4 Lesson 6 Transformations of inverse trigonometric functions	Chapter 4 Lesson 6 Evaluate inverse trigonometric functions	Chapter 4 Lesson 6 Evaluate compositions of trigonometric functions.

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4-5 Study Guide and Intervention

Graphing Other Trigonometric Functions

Tangent and Reciprocal Functions You can use the same techniques you learned for graphing the sine and cosine functions to graph the tangent function and the reciprocal trigonometric functions—cotangent, secant, and cosecant. The general form of the tangent function, which is similar to that of the sinusoidal functions, is

$$y = a \tan (bx + c) + d,$$

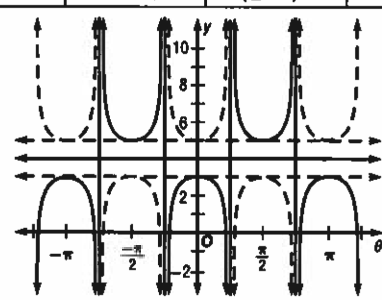
where a produces a vertical stretch or compression, b affects the period, c produces a horizontal translation, and d produces a vertical shift. The term *amplitude* does not apply to the tangent or cotangent functions because the heights of these functions are infinite.

Example **Locate the vertical asymptotes and sketch the graph of $y = \sec (2x + \pi) + 4$.**

The period of $y = \sec (2x + \pi) + 4$ is $\frac{2\pi}{|b|}$ or π. Since $c = \pi$, the phase shift is $-\frac{\pi}{2}$. Therefore, the vertical asymptotes are located every $\frac{\pi}{2}$ units at $-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$, and $\frac{5\pi}{4}$. The vertical shift is 4. Create a table including the relative maximum and minimum points for the period $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

Functions	Vertical Asymptote	Relative Minimum	Vertical Asymptote	Relative Maximum	Vertical Asymptote	Relative Minimum	Vertical Asymptote
$y = \sec x$	$x = -\frac{\pi}{2}$	(0, 1)	$x = \frac{\pi}{2}$	$(\pi, -1)$	$x = \frac{3\pi}{2}$	$(2\pi, 1)$	$x = \frac{5\pi}{2}$
$y = \sec (2x)$	$x = -\frac{\pi}{4}$	(0, 1)	$x = \frac{\pi}{4}$	$\left(\frac{\pi}{2}, -1\right)$	$x = \frac{3\pi}{4}$	$(\pi, 1)$	$x = \frac{5\pi}{4}$
$y = \sec (2x + \pi)$	$x = -\frac{3\pi}{4}$	$\left(-\frac{\pi}{2}, 1\right)$	$x = -\frac{\pi}{4}$	(0, −1)	$x = \frac{\pi}{4}$	$\left(\frac{\pi}{2}, 1\right)$	$x = \frac{3\pi}{4}$
$y = \sec (2x + \pi) + 4$	$x = -\frac{3\pi}{4}$	$\left(-\frac{\pi}{2}, 5\right)$	$x = -\frac{\pi}{4}$	(0, 3)	$x = \frac{\pi}{4}$	$\left(\frac{\pi}{2}, 5\right)$	$x = \frac{3\pi}{4}$

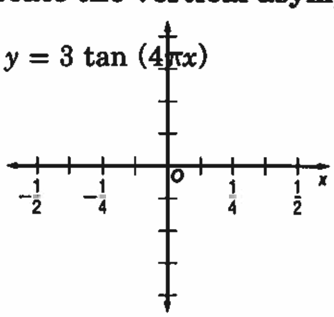
Graph one cycle on the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Then sketch one cycle to the left and right.



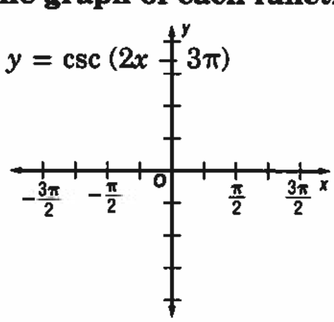
Exercises

Locate the vertical asymptotes and sketch the graph of each function.

1. $y = 3 \tan (4\pi x)$



2. $y = \csc (2x + 3\pi)$



4-5 Study Guide and Intervention *(continued)*

Graphing Other Trigonometric Functions

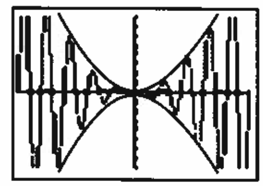
Damped Trigonometric Functions A damped trigonometric function is the product of a sinusoidal function of the form $y = \sin bx$ or $y = \cos bx$ and another function $y = f(x)$, called the damping factor. The graph of the product function is the oscillation of the sinusoidal function between the graphs of $y = f(x)$ and $y = -f(x)$. The resulting graph is called a damped wave, and the reduction in the amplitude is known as damped oscillation. Damped oscillation can occur as x approaches $\pm\infty$, as x approaches 0, or both.

Damped harmonic motion occurs when the amplitude of a function is damped due to friction over time. An object is in damped harmonic motion when the amplitude is determined by the function $a(t) = ke^{-ct}$. For $y = ke^{-ct} \sin \omega t$ and $y = ke^{-ct} \cos \omega t$, where $c > 0$, k is the displacement, c is the damping constant, t is time, and ω is the period.

Example **Identify the damping factor $f(x)$ of $y = \frac{1}{2}x^2 \sin 5x$. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.**

The function $y = \frac{1}{2}x^2 \sin 5x$ is the product of the functions $y = \frac{1}{2}x^2$ and $y = \sin 5x$. Therefore, the damping factor is $f(x) = \frac{1}{2}x^2$.

The amplitude of the function is decreasing as x approaches zero.



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-8, 8]$ scl: 1

Exercises

Identify the damping factor $f(x)$ of each function. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.

1. $y = 3x \sin 2x$

2. $y = \frac{1}{2}x \cos 4x$

3. A guitar string is plucked at a distance of 0.9 centimeter above its resting position and then released, causing vibration. The damping constant of the guitar string is 1.8, and the note produced has a frequency of 185 cycles per second.

a. Write a trigonometric function that models the motion of the string.

b. Determine the amount of time t that it takes the string to be damped so that $-0.28 \le y \le 0.28$.

Lesson 4-5

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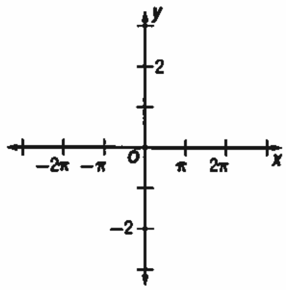
4-5 Practice

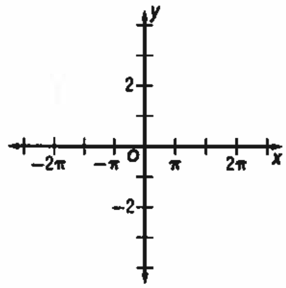
Graphing Other Trigonometric Functions

Locate the vertical asymptotes, and sketch the graph of each function.

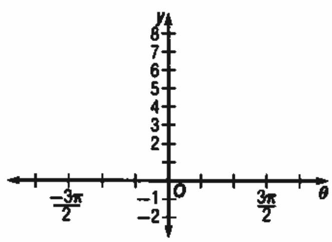
1. $y = -3 \tan x$



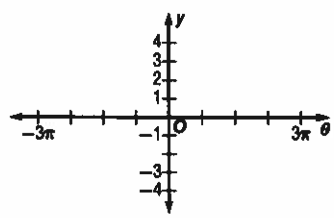
2. $y = -2 \cot\left(2x + \frac{\pi}{3}\right)$



3. $y = \csc x + 3$



4. $y = \sec\left(\frac{x}{3} + \pi\right) - 1$



Identify the damping factor $f(x)$ of the function. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.

5. $y = \frac{1}{2}x \cos 2x$

6. $y = -\frac{3}{2}x \sin \frac{\pi x}{2}$

7. MUSIC A guitar string is plucked at a distance of 0.6 centimeter above its resting position and then released, causing vibration. The damping constant of the guitar string is 1.8, and the note produced has a frequency of 105 cycles per second.

a. Write a trigonometric function that models the motion of the string.

b. Determine the amount of time t that it takes the string to be damped so that $-0.24 \leq y \leq 0.24$.

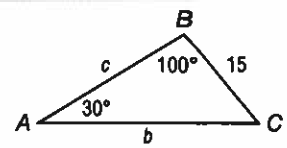
4-7 Study Guide and Intervention

The Law of Sines and the Law of Cosines

Solve Oblique Triangles The **Law of Sines** can be used to solve an oblique triangle when given the measures of two angles and a nonincluded side (AAS), two angles and the included side (ASA), or two sides and a nonincluded angle (SSA).

The **Law of Cosines** can be used to solve an oblique triangle when given the measures of three sides (SSS) or the measures of two sides and their included angle (SAS).

Example **Solve △*ABC*. Round side lengths to the nearest tenth and angle measures to the nearest degree.**



Because two angles are given,
$C = 180° - (100° + 30°)$ or $50°$.

Use the Law of Sines to find b and c.

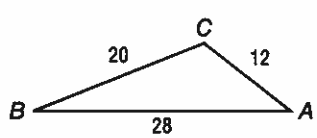
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines	$\frac{\sin A}{a} = \frac{\sin C}{c}$
$\frac{\sin 30°}{15} = \frac{\sin 100°}{b}$	Substitution	$\frac{\sin 30°}{15} = \frac{\sin 50°}{c}$
$b \sin 30° = 15 \sin 100°$	Cross products	$c \sin 30° = 15 \sin 50°$
$b = \frac{15 \sin 100°}{\sin 30°}$	Divide each side by sin 30°.	$c = \frac{15 \sin 50°}{\sin 30°}$
$b \approx 29.5$	Use a calculator.	$c \approx 23.0$

Therefore, $b \approx 29.5$, $c \approx 23.0$, and $C = 50°$.

Exercises

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1.



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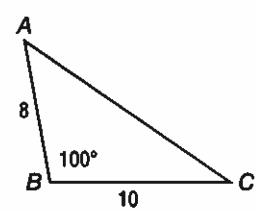

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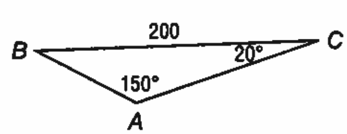

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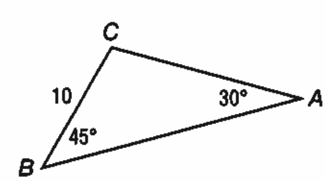


4-7 Study Guide and Intervention *(continued)*

The Law of Sines and the Law of Cosines

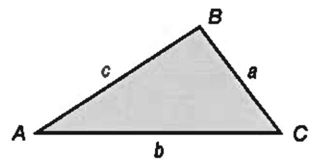
Find Areas of Oblique Triangles When the measures of all three sides of an oblique triangle are known, Heron's Formula can be used to find the area of the triangle.

Heron's Formula

If the measures of the sides of $\triangle ABC$ are a, b, and c, then the area of the triangle is

$\sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \frac{1}{2}(a+b+c)$.



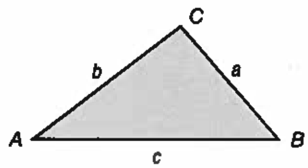
When two sides and the included angle of a triangle are known, the area is one-half the product of the lengths of the two sides and the sine of the included angle.

Area of a Triangle Given SAS

Area $= \frac{1}{2}bc \sin A$

Area $= \frac{1}{2}ac \sin B$

Area $= \frac{1}{2}ab \sin C$



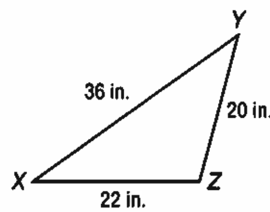
Example **Find the area of $\triangle XYZ$ to the nearest tenth.**

The value of s is $\frac{1}{2}(20 + 22 + 36)$ or 39.

Area $= \sqrt{s(s-x)(s-y)(s-z)}$ — Heron's Formula

$= \sqrt{39(39-20)(39-22)(39-36)}$ — $s = 39$, $x = 20$, $y = 22$, and $z = 36$

$= \sqrt{37{,}791}$ or about 194.4 in^2 — Simplify.



Exercises

Use Heron's Formula to find the area of each triangle. Round to the nearest tenth.

1. $\triangle ABC$ if $a = 14$ ft, $b = 9$ ft, $c = 8$ ft

2. $\triangle FGH$ if $f = 8$ in., $g = 9$ in., $h = 3$ in.

3. $\triangle MNP$ if $m = 3$ yd, $n = 4.6$ yd, $p = 5$ yd

4. $\triangle XYZ$ if $x = 8$ cm, $y = 12$ cm, $z = 13$ cm

Find the area of each triangle to the nearest tenth.

5. $\triangle RST$ if $R = 50°$, $s = 12$ yd, $t = 14$ yd

6. $\triangle MNP$ if $n = 14$ ft, $P = 110°$, $N = 25°$

7. $\triangle DEF$ if $d = 15$ ft, $E = 135°$, $f = 18$ ft

8. $\triangle JKL$ if $j = 4.3$ m, $l = 3.9$ m, $K = 82°$

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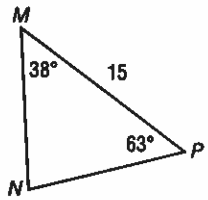
4-7 Practice

The Law of Sines and the Law of Cosines

Solve each triangle. Round to the nearest tenth if necessary.

1.



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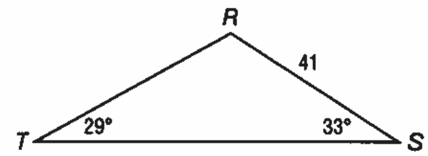

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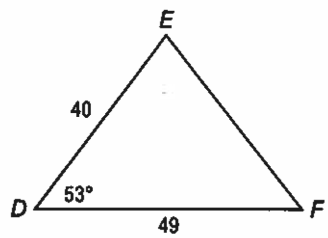

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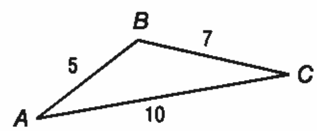
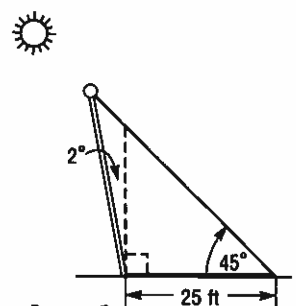


5. STREET LIGHTING A lamp post tilts toward the Sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamp post is 45°. Find the length of the lamp post.



Use Heron's Formula to find the area of each triangle. Round to the nearest tenth.

6. $\triangle ABC$ if $a = 5$ ft, $b = 12$ ft, $c = 13$ ft

7. $\triangle FGH$ if $f = 11$ in., $g = 13$ in., $h = 16$ in.

8. $\triangle MNP$ if $m = 8$ yd, $n = 3.6$ yd, $p = 5.2$ yd

9. $\triangle XYZ$ if $x = 12$ cm, $y = 10$ cm, $z = 15.8$ cm

Find the area of each triangle to the nearest tenth.

10. $\triangle RST$ if $R = 115°$, $s = 15$ yd, $t = 20$ yd

11. $\triangle MNP$ if $n = 4$ ft, $P = 69°$, $N = 37°$

12. $\triangle DEF$ if $d = 2$ ft, $E = 85°$, $F = 19°$

13. $\triangle JKL$ if $j = 68$ cm, $l = 110$ cm, $K = 42.5°$

NAME ____________________ DATE ________ PERIOD ________

4-6 Study Guide and Intervention

Inverse Trigonometric Functions

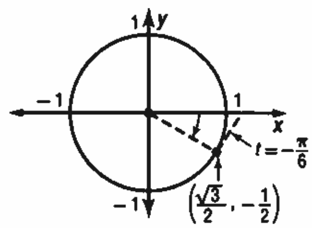
Inverse Trigonometric Functions When restricted to a certain domain, the sine, cosine, and tangent functions have inverse functions known as the arcsine, arccosine, and arctangent functions, respectively.

Inverse Trigonometric Functions	
Inverse Sine of x	$y = \sin^{-1} x$ or $y = \arcsin x$ if and only if $\sin y = x$, for $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.
Inverse Cosine of x	$y = \cos^{-1} x$ or $y = \arccos x$ if and only if $\cos y = x$, for $-1 \le x \le 1$ and $0 \le y \le \pi$.
Inverse Tangent of x	$y = \tan^{-1} x$ or $y = \arctan x$ if and only if $\tan y = x$, for $-\infty \le x \le \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Examples **Find the exact value of each expression, if it exists.**

1. $\sin^{-1}\left(-\frac{1}{2}\right)$

Find a point on the unit circle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a y-coordinate of $-\frac{1}{2}$. When $t = -\frac{\pi}{6}$, $\sin t = -\frac{1}{2}$. Therefore, $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.



2. $\cos^{-1} 4$

Because the domain of the inverse cosine function is $[-1, 1]$ and $4 > 1$, there is no angle with a cosine of 4. Therefore, the value of $\cos^{-1} 4$ does not exist.

Exercises

Find the exact value of each expression, if it exists.

1. arctan 0

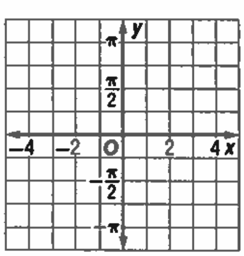
2. $\arcsin \frac{\sqrt{3}}{2}$

3. $\cos^{-1} \frac{\sqrt{2}}{2}$

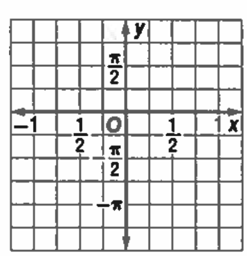
4. $\tan^{-1}(-1)$

Sketch the graph of the function.

5. $y = \arcsin 2x - 1$



6. $y = \tan^{-1}(x - 1)$



4-6 Study Guide and Intervention *(continued)*

Inverse Trigonometric Functions

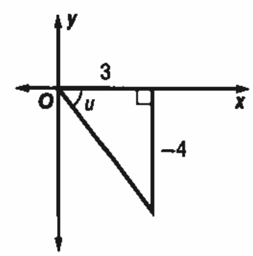
Compositions of Trigonometric Functions Because the domains of the trigonometric functions are restricted to obtain the inverse trigonometric functions, the composition of a trigonometric function and its inverse does not follow the rules that you learned in Lesson 1-7. The properties that apply to trigonometric functions and their inverses are summarized below.

Inverse Properties of Trigonometric Functions	
$f(f^{-1}(x)) = x$	$f^{-1}(f(x)) = x$
If $-1 \le x \le 1$, then $\sin(\sin^{-1} x) = x$.	If $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $\sin^{-1}(\sin x) = x$.
If $-1 \le x \le 1$, then $\cos(\cos^{-1} x) = x$.	If $0 \le x \le \pi$, then $\cos^{-1}(\cos x) = x$.
If $-\infty < x < \infty$, then $\tan(\tan^{-1} x) = x$.	If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $\tan^{-1}(\tan x) = x$.

Example **Find the exact value of $\cos\left[\tan^{-1} -\frac{4}{3}\right]$.**

To simplify the expression, let $u = \tan^{-1}\left(-\frac{4}{3}\right)$, so $\tan u = -\frac{4}{3}$.

Because the tangent function is negative in Quadrants II and IV and the domain of the inverse tangent function is restricted to Quadrants I and IV, u must lie in Quadrant IV.



Using the Pythagorean Theorem, you can find that the length of the hypotenuse is 5. Now solve for $\cos u$.

$\cos u = \frac{\text{adj}}{\text{hyp}}$ Cosine function

$= \frac{3}{5}$ adj = 3 and hyp = 5

So, $\cos\left[\tan^{-1} -\frac{4}{3}\right] = \frac{3}{5}$.

Exercises

Find the exact value of each expression, if it exists.

1. $\sin\left(\sin^{-1} -\frac{3}{4}\right)$

2. $\cos^{-1}\left(\cos \frac{\pi}{2}\right)$

3. $\tan^{-1}\left(\tan \frac{3\pi}{2}\right)$

4. $\sin^{-1}\left(\cos \frac{\pi}{6}\right)$

5. $\cos\left(\arcsin \frac{1}{2}\right)$

6. $\tan\left(\arcsin -\frac{1}{2}\right)$

7. $\cos(\arccos 2)$

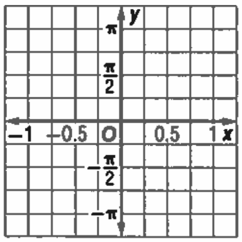
8. $\cos\left(\arctan -\frac{\sqrt{3}}{3}\right)$

4-6 Practice

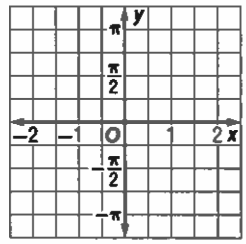
Inverse Trigonometric Functions

Sketch the graph of each function.

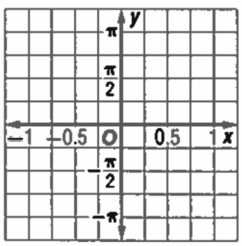
1. $y = \arccos 3x$



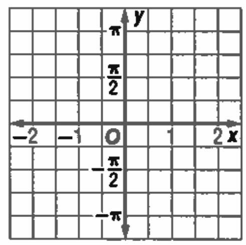
2. $y = \arctan x + 1$



3. $y = \sin^{-1} 3x$



4. $y = \tan^{-1} 3x$



Find the exact value of each expression, if it exists.

5. $\arcsin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

6. $\cos^{-1}\left(\cos \frac{\pi}{3}\right)$

7. $\tan\left(-\frac{3\pi}{2}\right)$

8. $\sin^{-1}\left(\cos \frac{\pi}{3}\right)$

9. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

10. $\arcsin\left(-\frac{1}{2}\right)$

11. $\tan\left(\sin^{-1} 1 - \cos^{-1} \frac{1}{2}\right)$

12. $\sin\left(\arctan -\frac{\sqrt{3}}{3}\right)$

13. ART Hans purchased a painting that is 30 inches tall that will hang 8 inches above the fireplace. The top of the fireplace is 55 inches from the floor.

a. Write a function modeling the maximum viewing angle θ for the distance d for Hans if his eye-level when sitting is 2.5 feet above the ground.

b. Determine the distance that corresponds to the maximum viewing angle.

Write each trigonometric expression as an algebraic expression of x.

14. $\sin(\arccos x)$

15. $\tan(\sin^{-1} x)$