High School Mathematics

PreCal
2019-2020

Instructional Packet
Set II

Students:

Work at your own pace. Contact your teacher for support as needed.

C & I Department

3 Student-Built Glossary

This is an alphabetical list of key vocabulary terms you will learn in Chapter 3. As you study this chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Precalculus Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
algebraic function		
common logarithm		
continuous compound interest		
exponential function		
linearize		
logarithm (LOG-uh-rith-um)		
logarithmic function with base *b*		
logistic growth function (lah-JIS-tik)		

(continued on the next page)

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3 Student-Built Glossary

Vocabulary Term	Found on Page	Definition/Description/Example
natural base		
natural logarithm		
transcendental function (tran-sen-DEN-tal)		

3 Anticipation Guide

Exponential and Logarithmic Functions

Step 1 *Before you begin Chapter 3*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. For most real-world applications involving exponential functions, the most commonly used base is not 2 or 10 but an irrational number *e*.	
	2. Logarithmic functions and exponential functions are inverses of each other.	
	3. A logarithm with base 10 or $\log_{10}$ is called a natural logarithm.	
	4. A logarithm with base *e* or $\log_e$ is called a common logarithm and is denoted by ln.	
	5. Exponential functions have the one-to-one property.	
	6. Logarithmic functions do not have the one-to-one property.	
	7. A logistic function is shown on a graph that changes rapidly and then has a horizontal asymptote.	
	8. Exponential models apply to any situation where the change is proportional to the initial size of the quantity being considered.	

Step 2 *After you complete Chapter 3*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

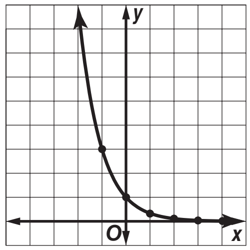
3-1 Study Guide and Intervention

Exponential Functions

Exponential Functions An **exponential function** with base b has the form $f(x) = ab^x$, where x is any real number and a and b are real number constants such that $a \neq 0$, b is positive, and $b \neq 1$. If $b > 1$, then the function is *exponential growth*. If $0 < b < 1$, then the function is *exponential decay*.

Example **Sketch and analyze the graph of $f(x) = \left(\frac{1}{3}\right)^x$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.**

x	–3	–2	–1	0	1	2	3
f(x)	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$



Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Intercept: $(0, 1)$ Asymptote: x-axis

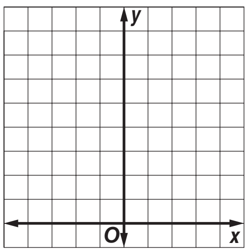
End behavior: $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

Decreasing: $(-\infty, \infty)$

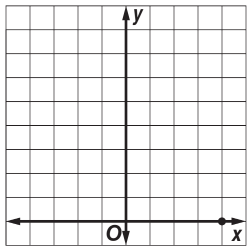
Exercises

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1. $h(x) = 2^{x-1} + 1$



2. $k(x) = e^{-2x}$



Lesson 3-1

3-1 Study Guide and Intervention *(continued)*

Exponential Functions

Exponential Growth and Decay Many real-world situations can be modeled by exponential functions. One of the equations below may apply.

Exponential Growth or Decay	Continuous Exponential Growth or Decay	Compound Interest
$N = N_0(1 + r)^t$ N is the final amount, N_0 is the initial amount, r is the rate of growth or decay, and t is time.	$N = N_0e^{kt}$ N is the final amount, N_0 is the initial amount, k is the rate of growth or decay, t is time, and e is a constant.	$A = P\left[1 + \frac{r}{n}\right]^{nt}$ P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is compounded each year, and t is the number of years.

Example 1 **BIOLOGY A researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?**

$N = N_0(1 + r)^t$ Exponential Growth Formula

$= 100(1 + 0.25)^6$ $N_0 = 100$, $r = 0.25$, $t = 6$

≈ 381.4697266 Use a calculator.

There will be about 381 cells in the colony in 6 weeks.

Example 2 **FINANCIAL LITERACY Lance has a bank account that will allow him to invest \$1000 at a 5% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Lance's account balance be after 10 years?**

$A = Pe^{rt}$ Continuous Compound Interest Formula

$= 1000e^{(0.05)(10)}$ $P = 1000$, $r = 0.05$, and $t = 10$

≈ 1648.72 Simplify.

With continuous compounding, Lance's account balance after 10 years will be \$1648.72.

Exercises

1. **FINANCIAL LITERACY** Compare the balance after 10 years of a \$5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.

2. **ENERGY** In 2007, it is estimated that the United States used about 101,000 quadrillion thermal units. If U.S. energy consumption increases at a rate of about 0.5% annually, what amount of energy will the United States use in 2020?

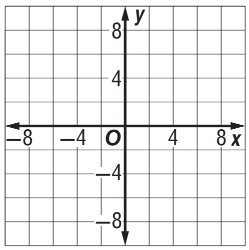
3. **BIOLOGY** The number of rabbits in a field showed an increase of 10% each month over the last year. If there were 10 rabbits at this time last year, how many rabbits are in the field now?

3-1 Practice

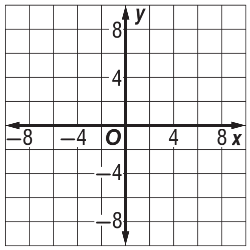
Exponential Functions

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1. $f(x) = 2^{x-1}$



2. $h(x) = -\frac{1}{5}e^{x} - 2$



3. DEMOGRAPHICS In 2000, the number of people in the United States was 281,421,906. The U.S. population is estimated to be growing at 0.88% annually.

a. Let t be the number of years since 2000. Write a function that models the annual growth in population in the U.S.

b. Predict the population in 2020 and 2030. Assume a steady rate of increase.

4. FINANCE Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial deposit is $1000 and the money is left in the account for 5 years.

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3-2 Study Guide and Intervention

Logarithmic Functions

Logarithmic Functions and Expressions

The inverse relationship between logarithmic functions and exponential functions can be used to evaluate logarithmic expressions.

If $b > 0$, $b \neq 1$, and $x > 0$, then

Logarithmic Form		**Exponential Form**
$\log_b x = y$	if and only if	$b^y = x$

In $\log_b x = y$, b is the base and y is the exponent. In $b^y = x$, b is the base and y is the exponent.

The following properties are also useful.

$\log_b 1 = 0$ $\qquad$ $\log_b b = 1$ $\qquad$ $\log_b b^x = x$ $\qquad$ $b^{\log_b x} = x, x > 0$

Example 1 **Evaluate each logarithm.**

a. $\log_5 \frac{1}{25}$

$\log_5 \frac{1}{25} = y$ $\qquad$ Let $\log_5 \frac{1}{25} = y$.

$5^y = \frac{1}{25}$ $\qquad$ Write in exponential form.

$5^y = 5^{-2}$ $\qquad$ $\frac{1}{25} = 5^{-2}$

$y = -2$ $\qquad$ Equality Prop. of Exponents

Therefore, $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

b. $\log_3 \sqrt{3}$

$\log_3 \sqrt{3} = y$ $\qquad$ Let $\log_3 \sqrt{3} = y$.

$3^y = \sqrt{3}$ $\qquad$ Write in exponential form.

$3^y = 3^{\frac{1}{2}}$ $\qquad$ $3^{\frac{1}{2}} = \sqrt{3}$

$y = \frac{1}{2}$ $\qquad$ Equality Prop. of Exponents

Therefore, $\log_3 \sqrt{3} = \frac{1}{2}$ because $3^{\frac{1}{2}} = \sqrt{3}$.

Example 2 **Evaluate each expression.**

a. $\ln e^7$

$\ln e^7 = 7$ $\qquad$ $\ln e^x = x$

b. $e^{\ln 5}$

$e^{\ln 5} = 5$ $\qquad$ $e^{\ln x} = x$

c. $10^{\log 13}$

$10^{\log 13} = 13$ $\qquad$ $10^{\log x} = x$

Exercises

Evaluate each logarithm.

1. $\log_7 7$

2. $10^{\log 5x}$

3. $3^{\log_3 2}$

4. $\log_6 36$

5. $\log_3 \frac{1}{81}$

6. $e^{\ln x^2}$

7. FINANCIAL LITERACY Ms. Dasilva has \$3000 to invest. She would like to invest in an account that compounds continuously at 6%. Use the formula $\ln A - \ln P = rt$, where A is the current balance, P is the original principal, r is the rate as a decimal, and t is the time in years. How long will it take for her balance to be \$6000?

3-2 Study Guide and Intervention *(continued)*

Logarithmic Functions

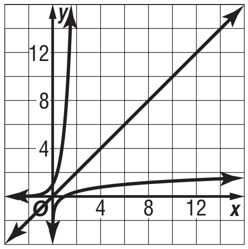
Graphs of Logarithmic Functions The inverse of $f(x) = b^x$ is called the logarithmic function with base b, or $f(x) = \log_b x$, and read *f of x equals the log base b of x*.

Example **Sketch and analyze the graph of $f(x) = \log_6 x$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.**

Create a table of values for the inverse of the function, the exponential function $f^{-1}(x) = 6^x$.

x	–2	–1	0	1	2
$f^{-1}(x)$	0.028	0.17	1	6	36

Since the functions are inverses, you can obtain the graph of $f(x)$ by plotting the points $\left(f^{-1}(x), x\right)$.



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

Asymptote: y-axis

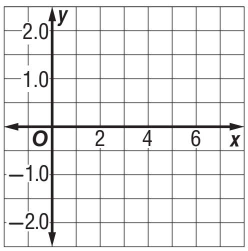
End behavior: $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Increasing: $(0, \infty)$

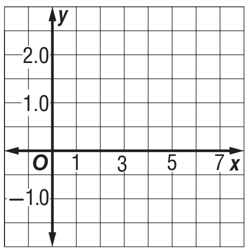
Exercises

Sketch and analyze the graph of each function below. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1. $g(x) = \log_3 x$



2. $h(x) = -\log_3 (x - 2)$



3-2 Practice

Logarithmic Functions

Evaluate each expression.

1. $\log_7 7^3$

2. $\log_{10} 0.001$

3. $\log_8 4096$

4. $2 \ln e^5$

5. $9^{\log_9 18}$

6. $\log_8 32$

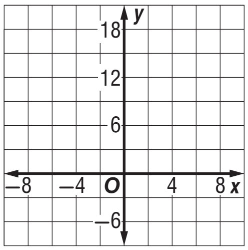
7. $\log_6 216$

8. $e^{\ln 0.014x}$

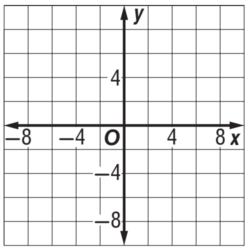
9. $\log_{12} 144$

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

10. $g(x) = 4^{-x+2}$

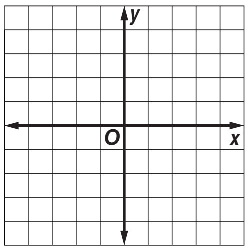


11. $g(x) = e^{2x+1}$



12. Use the graph of f to describe the transformation that results in the graph of g. Then sketch the graphs of f and g.

$$f(x) = \ln x,\ g(x) = \ln\left(\frac{x}{2}\right) - 2$$



13. INVESTMENTS The annual growth rate r for an investment can be found using $r = \frac{1}{t} \ln \frac{P}{P_0}$, where t is time in years, P is the present value, and P_0 is the original investment. An investment of $4000 was made in 2005 and had a value of $7500 in 2010. What was the average growth rate of the investment?

3-3 Study Guide and Intervention

Properties of Logarithms

Properties of Logarithms Since logarithms and exponents have an inverse relationship, they have certain properties that can be used to make them easier to simplify and solve.

If b, x, and y are positive real numbers, $b \neq 1$, and p is a real number, then the following statements are true.

- $\log_b xy = \log_b x + \log_b y$ — Product Property
- $\log_b \frac{x}{y} = \log_b x - \log_b y$ — Quotient Property
- $\log_b x^p = p \log_b x$ — Power Property

Example 1 **Evaluate $3 \log_2 8 + 5 \log_2 \frac{1}{2}$.**

$3 \log_2 8 + 5 \log_2 \frac{1}{2} = 3 \log_2 2^3 + 5 \log_2 2^{-1}$	$8 = 2^3$; $2^{-1} = \frac{1}{2}$
$= 3(3 \log_2 2) + 5(-\log_2 2)$	Power Property
$= 3(3)(1) + 5(-1)(1)$	$\log_x x = 1$
$= 4$	Simplify.

Example 2 **Expand $\ln \frac{8x^5}{3y^2}$.**

$\ln \frac{8x^5}{3y^2} = \ln 8x^5 - \ln 3y^2$	Quotient Property
$= \ln 8 + \ln x^5 - \ln 3 - \ln y^2$	Product Property
$= \ln 8 + 5 \ln x - \ln 3 - 2 \ln y$	Power Property

Exercises

1. Evaluate $2 \log_3 27 + 4 \log_3 \frac{1}{3}$.

Expand each expression.

2. $\log_3 \frac{5r^5}{\sqrt[3]{t^2}}$

3. $\log \frac{(a-2)(b+4)^6}{9(b-2)^5}$

Condense each expression.

4. $11 \log_9 (x - 3) - 5 \log_9 2x$

5. $\frac{3}{4} \ln (2h - k) + \frac{3}{5} \ln (2h + k)$

Lesson 3-3

3-3 Study Guide and Intervention *(continued)*

Properties of Logarithms

Change of Base Formula If the logarithm is in a base that needs to be changed to a different base, the **Change of Base Formula** is required.

For any positive real numbers a, b, and x, $a \neq 1$, $b \neq 1$, $\log_b x = \dfrac{\log_a x}{\log_a b}$.

Many non-graphing calculators cannot be used for logarithms that are not base e or base 10. Therefore, you will often use this formula, especially for scientific applications. Either of the following forms will provide the correct answer.

$$\log_b x = \frac{\log x}{\log b} \qquad \log_b x = \frac{\ln x}{\ln b}$$

Example **Evaluate each logarithm.**

a. $\log_2 7$

$\log_2 7 = \dfrac{\ln 7}{\ln 2}$ Change of Base Formula

≈ 2.81 Use a calculator.

b. $\log_{\frac{1}{3}} 10$

$\log_{\frac{1}{3}} 10 = \dfrac{\log 10}{\log \frac{1}{3}}$ Change of Base Formula

≈ -2.10 Use a calculator.

Exercises

Evaluate each logarithm.

1. $\log_{32} 631$

2. $\log_3 17$

3. $\log_7 1094$

4. $\log_6 94$

5. $\log_5 256$

6. $\log_9 712$

7. $\log_6 832$

8. $\log_{11} 47$

9. $\log_3 9$

10. $\log_8 256$

11. $\log_{12} 4302$

12. $\log_{0.5} 420$

3-3 Practice

Properties of Logarithms

Express each logarithm in terms of ln 10 and ln 3.

1. $\ln 300$

2. $\ln 27000$

3. $\ln \frac{10}{9}$

4. $\ln \frac{729}{10000}$

Expand each expression.

5. $\ln \frac{x+1}{\sqrt[4]{x-5}}$

6. $\ln \frac{x^2}{\sqrt{3x+2}}$

7. $\log_2 [(2x)^3(x+1)]$

8. $\log_8 [(4x+2)^3(x-4)]$

9. $\log_{13} \frac{3x^4}{\sqrt[3]{7x-3}}$

10. $\log_2 \frac{(x+1)^3}{\sqrt[3]{x+5}}$

Condense each expression.

11. $\frac{1}{2} \ln (3x - 5y) - \ln (4x + y)$

12. $3 \log_2 (5x+6) - \frac{1}{2} \log_2 (x-4)$

13. $2 - \log_7 6 - 2 \log_7 x$

14. $\log_3 8 + \log_3 x - 2 \log_3 (x+4)$

15. $\log y + \log 3 - \frac{1}{3} \log(x) + 2 \log z$

16. $\log_3 y + \log_3 x - \frac{1}{2} \log_3 x + 3 \log_3 z$

Evaluate each logarithm.

17. $\log_{\frac{1}{2}} \frac{1}{5}$

18. $\log_{100} 200$

19. $\log_{0.01} 4$

20. $\log_{0.24} 322$

21. $\log_6 24$

22. $\log_{\frac{1}{3}} 9.8$

23. SEISMOLOGY The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where R is the magnitude, I is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

a. Guam region, in 2008, $R = 6.7$

b. Macquarie Island region, in 2008, $R = 7.1$

3-4 Study Guide and Intervention

Exponential and Logarithmic Equations

Solve Exponential Equations

One-to-One Property of Exponential Functions: For $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

This property will help you solve exponential equations. For example, you can express both sides of the equation as an exponent with the same base. Then use the property to set the exponents equal to each other and solve. If the bases are not the same, you can *exponentiate* each side of an equation and use logarithms to solve the equation.

Example 1

a. Solve $4^{x-1} = 16^x$.

$4^{x-1} = 16^x$	Original equation
$4^{x-1} = (4^2)^x$	$16 = 4^2$
$4^{x-1} = 4^{2x}$	Power of a Power
$x - 1 = 2x$	One-to-One Property
$-1 = x$	Subtract x from each side.

b. Solve $e^{2x} - 3e^x + 2 = 0$.

$e^{2x} - 3e^x + 2 = 0$	Original equation
$u^2 - 3u + 2 = 0$	Write in quadratic form.
$(u - 2)(u - 1) = 0$	Factor.
$u = 2$ or $u = 1$	Solve.
$e^x = 2$ or $e^x = 1$	Substitute for u.
$x = \ln 2$ or 0	Take the natural logarithm of each side.

Example 2 **Solve each equation. Round to the nearest hundredth if necessary.**

a. $3^x = 19$

$\log 3^x = \log 19$	Take the log of both sides.
$x \log 3 = \log 19$	Power Property
$x = \dfrac{\log 19}{\log 3}$	Divide each side by log 3.
$x \approx 2.68$	Use a calculator. Check this solution in the original equation.

b. $e^{8x+1} - 6 = 1$

$e^{8x+1} = 7$	Add 6 to both sides.
$\ln e^{8x+1} = \ln 7$	Take the ln of both sides.
$(8x + 1) \ln e = \ln 7$	Power Property
$8x + 1 = \ln 7$	$\ln e = 1$
$8x = \ln 7 - 1$	Subtract 1 from each side.
$x = \dfrac{\ln 7 - 1}{8} \approx 0.12$	Divide by 8 and use a calculator.

Exercises

Solve each equation. Round to the nearest hundredth.

1. $9^x = 3^{3x-4}$

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

3. $4^{3x-2} = \frac{1}{2}^{2x}$

4. $2e^{2x} + 12e^x - 54 = 0$

5. $9^{2x} = 12$

6. $2.4e^{x-6} = 9.3$

7. $3^{2x} = 6^{x-1}$

8. $e^{19x} = 23$

3-4 Study Guide and Intervention *(continued)*

Exponential and Logarithmic Equations

Solve Logarithmic Equations

One-to-One Property of Exponential Functions For $b > 0$ and $b \neq 1$, $\log_b x = \log_b y$ if and only if $x = y$.

This property will help you solve logarithmic equations. For example, you can express both sides of the equation as a logarithm with the same base. Then convert both sides to exponential form, set the exponents equal to each other and solve.

Example 1 **Solve $2 \log_5 4x - 1 = 11$.**

$2 \log_5 4x - 1 = 11$	Original equation
$2 \log_5 4x = 12$	Add 1 to each side.
$\log_5 4x = 6$	Divide each side by 2.
$4x = 5^6$	Write in exponential form. (Use 5 as the base when exponentiating.)
$x = \frac{5^6}{4}$	Divide each side by 4.
$x = 3906.25$	Use a calculator.

Example 2 **Solve $\log_2(x - 6) = 5 - \log_2 2x$.**

$\log_2 (x - 6) = 5 - \log_2 2x$	Original equation
$\log_2 (x - 6) + \log_2 2x = 5$	Rearrange the logs.
$\log_2 (2x(x - 6)) = 5$	Product Property
$2x(x - 6) = 2^5$	Rewrite in exponential form.
$2x^2 - 12x - 32 = 0$	Expand.
$2(x - 8)(x + 2) = 0$	Factor.
$x = 8$ or -2	Solve.

CHECK

$x = -2$ $\log_2 (-2 - 6) = 5 - \log_2 [2(-2)]$ yields logs of negative numbers. Therefore, -2 is extraneous.

$x = 8$ $\log_2 (8 - 6) = 5 - \log_2 [2(8)]$

$\log_2 2 = 5 - \log_2 16$, which is true. Therefore, $x = 8$.

Exercises

Solve each logarithmic equation.

1. $\log 3x = \log 12$

2. $\log_{12} 2 + \log_{12} x = \log_{12} (x + 7)$

3. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

4. $\log_3 3x = \log_3 36$

5. $\log (16x + 2) + \log (20x - 2) = \log (319x^2 + 9x - 2)$

6. $\ln x + \ln (x + 16) = \ln 8 + \ln (x + 6)$

3-4 Practice

Exponential and Logarithmic Equations

Solve each equation.

1. $5^x = 125^{x-2}$

2. $\log_6 x + \log_6 9 = \log_6 54$

3. $\left(\frac{1}{9}\right)^{x+3} = 27^x$

4. $e^{2x} - e^x - 6 = 0$

5. $\log_x 64 = 3$

6. $\ln \frac{1}{e} = x$

7. $\ln (2x - 1) = \ln 16$

8. $3e^{4x} - 9e^{2x} - 15 = 0$

9. $\ln (x - 5) + \ln 4 = \ln x - \ln 2$

10. $4^{x+2} = 6^{-2x-3}$

11. $6e^{6x} - 17e^{3x} + 7 = 0$

12. $6 \ln (x + 2) - 3 = 21$

13. $4e^{2x} - 13e^x + 9 = 0$

14. $\log (2x + 1) + \log (x - 4) = \log (2x^2 - 4)$

15. $2^{-4x+1} = 3^{2x-3}$

16. $\log_5 (x + 4) + \log_5 x = \log_5 12$

17. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

18. $\ln 0.04x = -8$

Solve each equation. Round to the nearest hundredth.

19. $2^{9x} = 1210$

20. $4^{3x} = 1056$

21. $5^{x+3} - 4 = 19$

22. $3^{x-8} + 2 = 38$

23. $6^{2x-1} = 18$

24. $2^{3+2x} = 130$

25. BANKING Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is $A = Pe^{rt}$. If Ms. Cubbatz made the investment on January 1, 2005, and the account was worth \$12,000 on January 1, 2009, what was the original amount in the account?

26. FINANCIAL LITERACY If \$500 is deposited in a savings account providing an annual interest rate of 5.6% compounded quarterly, how long will it take for the account to be worth \$750?

4 Student-Built Glossary

This is an alphabetical list of key vocabulary terms you will learn in Chapter 4. As you study this chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Precalculus Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
amplitude		
angle of depression		
angle of elevation		
cosecant		
cosine		
coterminal angle		
frequency		
Law of Cosines		
Law of Sines		
period		

(continued on the next page)

NAME ______________________________ DATE ____________ PERIOD ____________

4 Student-Built Glossary

Vocabulary Term	Found on Page	Definition/Description/Example
phase shift		
radian		
reference angle		
secant		
sine		
standard position		
tangent		
unit circle		
vertical shift		

4 Anticipation Guide

Trigonometric Identities and Equations

Step 1 *Before you begin Chapter 4*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. If a trigonometric ratio is given in a quadrant, the remaining five trigonometric ratios can be found.	
	2. The Law of Sines can only be used with right triangles.	
	3. The area of a sector of a circle can be found if the radius and a central angle are known.	
	4. One full rotation on the unit circle is π radians.	
	5. The amplitude of a sinusodial function is the distance from the highest point to the lowest point.	
	6. The rate at which an object moves along a circular path is called its linear speed.	
	7. The period of a sinusoidal function refers to the number of cycles the function completes in a one unit interval.	
	8. Trigonometric functions have inverses.	
	9. The sine of an acute angle is the ratio of the measure of the side opposite the angle in a right triangle to the measure of the side adjacent to the angle.	
	10. Heron's formula can be used to find the perimeter of any triangle.	
	11. The graph of the sine function has vertical asymptotes at odd multiples of $\frac{\pi}{2}$.	
	12. A damped trigonometric function can be used to model the vibrations of a guitar string.	

Step 2 *After you complete Chapter 4*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

4-1 Study Guide and Intervention

Right Triangle Trigonometry

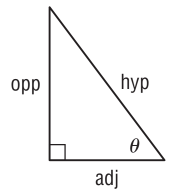
Values of Trigonometric Ratios The side lengths of a right triangle and a reference angle θ can be used to form six **trigonometric ratios** that define the **trigonometric functions** known as **sine**, **cosine**, and **tangent**. The **cosecant**, **secant**, and **cotangent** ratios are reciprocals of the sine, cosine, and tangent ratios, respectively. Therefore, they are known as **reciprocal functions**.

Let θ be an acute angle in a right triangle and the abbreviations opp, adj, and hyp refer to the lengths of the side opposite θ, the side adjacent to θ, and the hypotenuse, respectively.

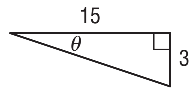
Then the six trigonometric functions of θ are defined as follows.

sine $(\theta) = \sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ cosine $(\theta) = \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ tangent $(\theta) = \tan\theta = \dfrac{\text{opp}}{\text{adj}}$

cosecant $(\theta) = \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ secant $(\theta) = \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ cotangent $(\theta) = \cot\theta = \dfrac{\text{adj}}{\text{opp}}$



Example **Find the exact values of the six trigonometric functions of θ.**



Use the Pythagorean Theorem to determine the length of the hypotenuse.

$15^2 + 3^2 = c^2$ $a = 15, b = 3$

$234 = c^2$ Simplify.

$c = \sqrt{234}$ or $3\sqrt{26}$ Take the positive square root.

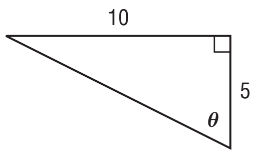
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3}{3\sqrt{26}}$ or $\dfrac{\sqrt{26}}{26}$ $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{15}{3\sqrt{26}}$ or $\dfrac{5\sqrt{26}}{26}$ $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{15}$ or $\dfrac{1}{5}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{3\sqrt{26}}{3}$ or $\sqrt{26}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{3\sqrt{26}}{15}$ or $\dfrac{\sqrt{26}}{5}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{15}{3}$ or 5

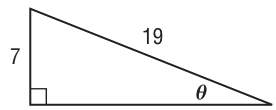
Exercises

Find the exact values of the six trigonometric functions of θ.

1.



2.



Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ.

3. $\sin\theta = \dfrac{3}{7}$

4. $\sec\theta = \dfrac{8}{5}$

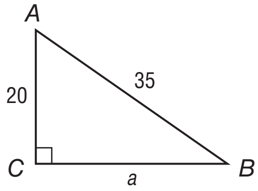
4-1 Study Guide and Intervention *(continued)*

Right Triangle Trigonometry

Solving Right Triangles To solve a right triangle means to find the measures of all of the angles and sides of the triangle. When the trigonometric value of an acute angle is known, the inverse of the trigonometric function can be used to find the measure of the angle.

Trigonometric Function	Inverse Trigonometric Function
$y = \sin x$	$x = \sin^{-1} y$ or $x = \arcsin y$
$y = \cos x$	$x = \cos^{-1} y$ or $x = \arccos y$
$y = \tan x$	$x = \tan^{-1} y$ or $x = \arctan y$

Example **Solve $\triangle ABC$. Round side measures to the nearest tenth and angle measures to the nearest degree.**



Because two lengths are given, you can use the Pythagorean Theorem to find that a is equal to $\sqrt{825}$ or about 28.7.
Find the measure of $\angle A$ using the cosine function.

$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ Cosine function

$\cos A = \frac{20}{35}$ Substitute $b = 20$ and $c = 35$.

$A = \cos^{-1} \frac{20}{35}$ Definition of inverse cosine

$A = 55.15009542$ Use a calculator.

Because A is now known, you can find B by subtracting A from 90°.

$55.15 + B = 90$ Angles A and B are complementary.

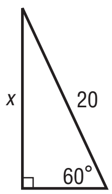
$B = 34.85°$ Subtract.

Therefore, $a \approx 28.7$, $A \approx 55°$, and $B \approx 35°$.

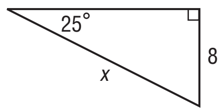
Exercises

Find the value of x. Round to the nearest tenth if necessary.

1.

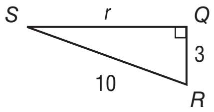


2.

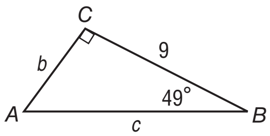


Solve each triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

3.



4.

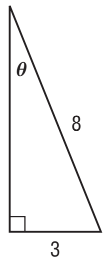


4-1 Practice

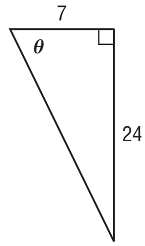
Right Triangle Trigonometry

Find the exact values of the six trigonometric functions of θ.

1.

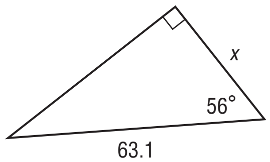


2.

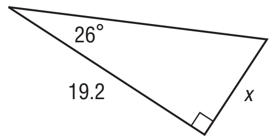


Find the value of x. Round to the nearest tenth, if necessary.

3.



4.

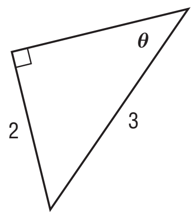


5. On a college campus, the library is 80 yards due east of the dormitory and the recreation center is due north of the library. The college is constructing a sidewalk from the dormitory to the recreation center. The sidewalk will be at a 56° angle with the current sidewalk between the dormitory and the library. To the nearest yard, how long will the new sidewalk be?

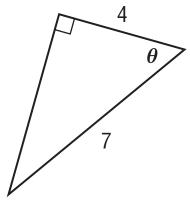
6. If $\cot A = 8$, find the exact values of the remaining trigonometric functions for the acute angle A.

Find the measure of angle θ. Round to the nearest degree, if necessary.

7.

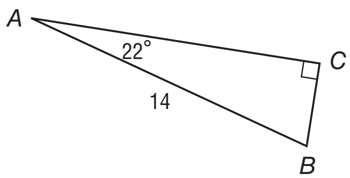


8.

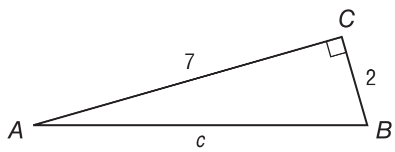


Solve each triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

9.



10.



11. SWIMMING The swimming pool at Perris Hill Plunge is 50 feet long and 25 feet wide. If the bottom of the pool is slanted so that the water depth is 3 feet at the shallow end and 15 feet at the deep end, what is the angle of elevation at the bottom of the pool?

4-2 Study Guide and Intervention

Degrees and Radians

Angles and Their Measures One complete rotation can be represented by 360° or 2π radians. Thus, the following formulas can be used to relate **degree** and **radian** measures.

Degree/Radian Coversion Rules	
$1° = \frac{\pi}{180}$ radians	$1 \text{ radian} = \left(\frac{180}{\pi}\right)°$

If two angles have the same initial and terminal sides, but different measures, they are called **coterminal angles**.

Example **Write each degree measure in radians as a multiple of π and each radian measure in degrees.**

a. 36°

$36° = 36°\left(\frac{\pi \text{ radians}}{180°}\right)$ Multiply by $\frac{\pi \text{ radians}}{180°}$.

$= \frac{\pi}{5}$ radians or $\frac{\pi}{5}$ Simplify.

b. $-\frac{17\pi}{3}$

$-\frac{17\pi}{3} = -\frac{17\pi}{3}$ radians Multiply by $\frac{180°}{\pi \text{ radians}}$.

$= -\frac{17\pi}{3} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right) = -1020°$ Simplify.

Exercises

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

1. $-250°$

2. $6°$

3. $-145°$

4. $870°$

5. $18°$

6. $-820°$

7. 4π

8. $\frac{13\pi}{30}$

9. -1

10. $\frac{3\pi}{16}$

11. -2.56

12. $-\frac{7\pi}{9}$

Identify all angles that are coterminal with the given angle.

13. $-\frac{\pi}{2}$

14. $135°$

15. $\frac{5\pi}{3}$

4-2 Study Guide and Intervention *(continued)*

Degrees and Radians

Applications with Angle Measure The rate at which an object moves along a circular path is called its **linear speed**. The rate at which the object rotates about a fixed point is called its **angular speed**.

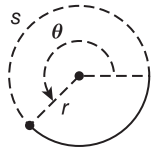
Suppose an object moves at a constant speed along a circular path of radius r.

If s is the arc length traveled by the object during time t, then the object's *linear speed* v is given by

$$v = \frac{s}{t}.$$

If θ is the angle of rotation (in radians) through which the object moves during time t, then the *angular speed* ω of the object is given by

$$\omega = \frac{\theta}{t}.$$



Example **Determine the angular speed and linear speed if 8.2 revolutions are completed in 3 seconds and the distance from the center of rotation is 7 centimeters. Round to the nearest tenth.**

The angle of rotation is $8.2 \times 2\pi$ or 16.4π radians.

$\omega = \frac{\theta}{t}$	Angular speed
$= \frac{16.4\pi}{3}$	$\theta = 16.4\pi$ radians and $t = 3$ seconds
≈ 17.17403984	Use a calculator.

Therefore, the angular speed is about 17.2 radians per second.

The linear speed is $\frac{r\theta}{t}$.

$v = \frac{s}{t}$	Linear speed
$= \frac{r\theta}{t}$	$s = r\theta$
$= \frac{7(16.4\pi)}{3}$	$r = 7$ centimeters, $\theta = 16.4\pi$ radians, and $t = 3$ seconds
$= 120.218278877$	Use a calculator.

Therefore, the linear speed is about 120.2 centimeters per second.

Exercises

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

1. $\omega = 2.7$ rad/s

2. $\omega = \frac{4}{3}\pi$ rad/hr

3. $\omega = \frac{3}{2}\pi$ rad/min

4. $v = 24.8$ m/s, 120 rev/min

5. $v = 118$ ft/min, 3.6 rev/s

6. $v = 256$ in./h, 0.5 rev/min

Lesson 4-2

4-2 Practice

Degrees and Radians

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

1. 28.955

2. -57.3278

3. $32\ \ 28' 10''$

4. $-73\ \ 14' 35''$

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

5. 25°

6. 130°

7. $\frac{3\pi}{4}$

8. $\frac{5\pi}{3}$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

9. 43°

10. $-\frac{7\pi}{4}$

Find the length of the intercepted arc with the given central angle measure in a circle of the given radius. Round to the nearest tenth.

11. 30°, $r = 8$ yd

12. $\frac{7\pi}{6}$, $r = 10$ in.

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

13. $\omega = \frac{4}{5}\pi$ rad/s

14. $v = 32$ m/s, 100 rev/min

15. On a game show, a contestant spins a wheel. The angular speed of the wheel was $\omega = \frac{\pi}{3}$ radians per second. If the wheel maintained this rate, what would be the rotation in revolutions per minute?

Find the area of each sector.

16. $\theta = \frac{\pi}{6}$, $r = 14$ in.

17. $\theta = \frac{7\pi}{4}$, $r = 4$ m

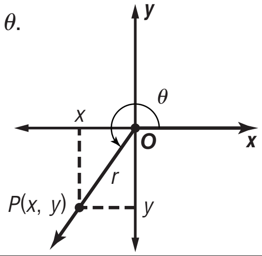
4-3 Study Guide and Intervention

Trigonometric Functions on the Unit Circle

Trigonometric Functions of Any Angle The definitions of the six trigonometric functions may be extended to include any angle as shown below.

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ. Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y},\ y \neq 0$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x},\ x \neq 0$
$\tan\theta = \frac{y}{x},\ x \neq 0$	$\cot\theta = \frac{x}{y},\ y \neq 0$



You can use the following steps to find the value of a trigonometric function of any angle θ.

1. Find the reference angle θ'.

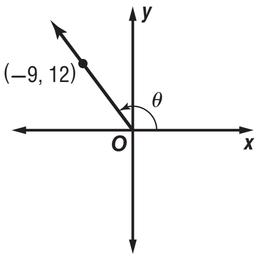
2. Find the value of the trigonometric function for θ'.

3. Use the quadrant in which the terminal side of θ lies to determine the sign of the trigonometric function value of θ.

Example **Let (−9, 12) be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ.**

Use the values of x and y to find r.

$r = \sqrt{x^2 + y^2}$	Pythagorean Theorem
$= \sqrt{(-9)^2 + 12^2}$	$x = -9$ and $y = 12$
$= \sqrt{225}$ or 15	Take the positive square root.



Use $x = -9$, $y = 12$, and $r = 15$ to write the six trigonometric ratios.

$\sin\theta = \frac{y}{r} = \frac{12}{15}$ or $\frac{4}{5}$ $\qquad \cos\theta = \frac{x}{r} = \frac{-9}{15}$ or $-\frac{3}{5}$ $\qquad \tan\theta = \frac{y}{x} = \frac{12}{-9}$ or $-\frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{15}{12}$ or $\frac{5}{4}$ $\qquad \sec\theta = \frac{r}{x} = \frac{15}{-9}$ or $-\frac{5}{3}$ $\qquad \cot\theta = \frac{x}{y} = \frac{-9}{12}$ or $-\frac{3}{4}$

Exercises

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (2, −5) **2.** (12, 4) **3.** (−3, −8)

Find the exact value of each expression.

4. $\sin\frac{5\pi}{3}$ **5.** csc 210° **6.** cot (−315°)

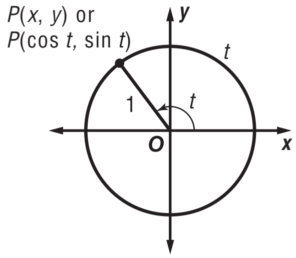
4-3 Study Guide and Intervention *(continued)*

Trigonometric Functions on the Unit Circle

Trigonometric Functions on the Unit Circle You can use the unit circle to find the values of the six trigonometric functions for θ. The relationships between θ and the point $P(x, y)$ on the unit circle are shown below.

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$\sin t = y$	$\cos t = x$	$\tan t = \frac{y}{x}, x \neq 0$
$\csc t = \frac{1}{y}, y \neq 0$	$\sec t = \frac{1}{x}, x \neq 0$	$\cot t = \frac{x}{y}, y \neq 0$



Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.

Example **Find the exact value of tan $\frac{5\pi}{3}$. If undefined, write *undefined*.**

$\frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit cirle.

$\tan t = \frac{y}{x}$ — Definition of tan t

$\tan \frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ — $x = \frac{1}{2}$ and $y = -\frac{\sqrt{3}}{2}$ when $t = \frac{5\pi}{3}$

$\tan \frac{5\pi}{3} = -\sqrt{3}$ — Simplify.

Exercises

Find the exact value of each expression. If undefined, write *undefined*.

1. $\tan \frac{\pi}{2}$

2. $\sec -\frac{3\pi}{4}$

3. $\cos \frac{7\pi}{6}$

4. $\sin \frac{5\pi}{4}$

5. $\cot \frac{4\pi}{3}$

6. $\csc -\frac{5\pi}{3}$

7. $\tan -60°$

8. $\cot 270°$

4-3 Practice

Trigonometric Functions on the Unit Circle

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. $(-1, 5)$

2. $(7, 0)$

3. $(-3, -4)$

4. $(1, -2)$

5. $(-3, 1)$

6. $(2, -4)$

Sketch each angle. Then find its reference angle.

7. $330°$

8. $-\frac{3\pi}{4}$

9. $\frac{7\pi}{6}$

10. $\frac{7\pi}{4}$

11. $135°$

12. $-\frac{\pi}{3}$

Find the exact value of each expression. If undefined, write *undefined*.

13. $\csc 90°$

14. $\tan 270°$

15. $\sin(-90°)$

16. $\cos \frac{3\pi}{2}$

17. $\sec\left(-\frac{\pi}{4}\right)$

18. $\cot \frac{5\pi}{6}$

19. PENDULUMS The angle made by the swing of a pendulum and its vertical resting position can be modeled by $\theta = 3 \cos \pi t$, where t is time measured in seconds and θ is measured in radians. What is the angle made by the pendulum after 6 seconds?

NAME ______________________ DATE __________ PERIOD __________

4-4 Study Guide and Intervention

Graphing Sine and Cosine Functions

Transformations of Sine and Cosine Functions A sinusoid is a transformation of the graph of the sine function. The general form of the sinusoidal functions sine and cosine are $y = a \sin (bx + c) + d$ or $y = a \cos (bx + c) + d$. The graphs of $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$ have the following characteristics.

Amplitude $= |a|$ Period $= \frac{2\pi}{|b|}$ Frequency $= \frac{|b|}{2\pi}$ or $\frac{1}{\text{period}}$

Phase Shift $= -\frac{c}{|b|}$ Vertical Shift $= d$ Midline $y = d$

Example **State the amplitude, period, frequency, phase shift, and vertical shift of $y = -2 \cos \left(\frac{x}{4} - \frac{\pi}{3}\right) + 2$. Then graph two periods of the function.**

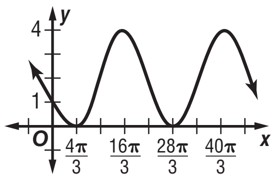
Amplitude $= |a| = |-2| = 2$

Period $= \frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{1}{4}\right|}$ or 8π

Frequency $= \frac{|b|}{2\pi} = \frac{\left|\frac{1}{4}\right|}{2\pi}$ or $\frac{1}{8\pi}$

Phase Shift $= -\frac{c}{|b|} = \frac{-\left(-\frac{\pi}{3}\right)}{\left|\frac{1}{4}\right|}$ or $\frac{4\pi}{3}$

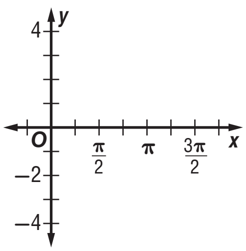
Vertical Shift $= d$ or 2



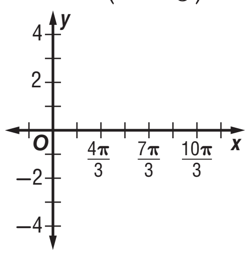
Exercises

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

1. $y = 3 \sin (2x + \pi)$



2. $y = \cos \left(x - \frac{\pi}{3}\right) + 2$



State the frequency and midline of each function.

3. $y = 3 \sin \left(\frac{1}{2}x + \frac{4\pi}{3}\right) + 1$

4. $y = \cos (3x) - 2$

Write a sine function with the given characteristics.

5. amplitude $= 2$, period $= 4$, phase shift $= \frac{1}{2}$, vertical shift $= 4$

6. amplitude $= 1.2$, phase shift $= \frac{3\pi}{2}$, vertical shift $= 1$

4-4 Study Guide and Intervention *(continued)*

Graphing Sine and Cosine Functions

Applications of Sinusoidal Functions You can use sinusoidal functions to solve certain application problems.

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan, in degrees Fahrenheit (°F). Write a sinusoidal function that models the average monthly temperatures as a function of time x, where $x = 1$ represents January.**

Month	Temperature
Jan	30°
Feb	34°
Mar	45°
Apr	59°
May	71°
June	80°
July	84°
Aug	81°
Sept	74°
Oct	62°
Nov	48°
Dec	35°

The data can be modeled by a sinusoidal function of the form $y = a \sin (bx + c) + d$. Find the maximum M and minimum m values of the data, and use these values to find a, b, c, and d.

$a = \frac{1}{2}(M - m)$ Amplitude formula

$= \frac{1}{2}(84 - 30)$ or 27 $M = 84$ and $m = 30$

$d = \frac{1}{2}(M + m)$ Vertical shift formula

$= \frac{1}{2}(84 + 30)$ or 57 $M = 84$ and $m = 30$

Period $= 2(x_{max} - x_{min})$ x_{max} = July or month 7 and

$= 2(7 - 1)$ or 12 x_{min} = January or month 1

$|b| = \frac{2\pi}{12}$ or $\frac{\pi}{6}$ $|b| = \frac{2\pi}{\text{period}}$

Phase shift $= -\frac{c}{|b|}$ Phase shift formula

$4 = -\frac{c}{\frac{\pi}{6}}$ Phase shift = 4 and $|b| = \frac{\pi}{6}$

$c = -\frac{2\pi}{3}$ Solve for c.

Therefore, $y = 27 \sin \left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.

Exercise

1. **MUSEUM ATTENDANCE** The table gives the number of visitors in thousands to a museum for each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Visitors	10	8	11	15	24	30	32	29

a. Write a trigonometric function that models the monthly attendance at the museum using $x = 1$ to represent January.

b. According to your model, how many people should the museum expect to visit during October?

Lesson 4-4

NAME ____________________ DATE ____________ PERIOD ____________

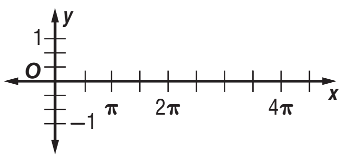
4-4 Practice

Graphing Sine and Cosine Functions

Describe how the graphs of $f(x)$ and $g(x)$ are related. Then find the amplitude of $g(x)$ and sketch two periods of both functions on the same coordinate axes.

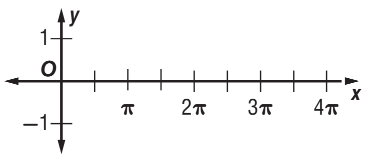
1. $f(x) = \sin x$

$g(x) = \frac{1}{3} \sin x$



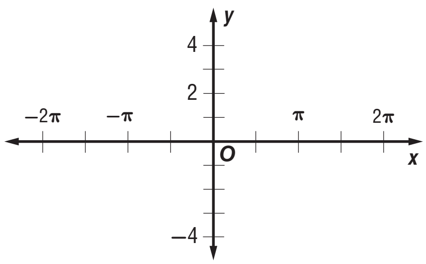
2. $f(x) = \cos x$

$g(x) = -\frac{1}{4} \cos x$

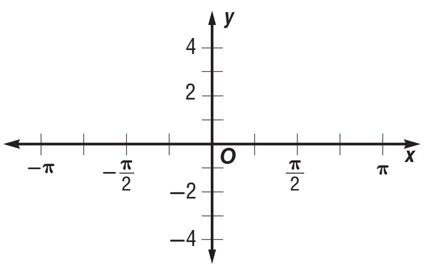


State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

3. $y = 2 \sin \left(x + \frac{\pi}{2}\right) - 3$



4. $y = \frac{1}{2} \cos (2x - \pi) + 2$



Write a sinusoidal function with the given amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = -10

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

7. MUSIC A piano tuner strikes a tuning fork note A above middle C and sets in motion vibrations that can be modeled by $y = 0.001 \sin 880t\pi$. Find the amplitude and period of the function.