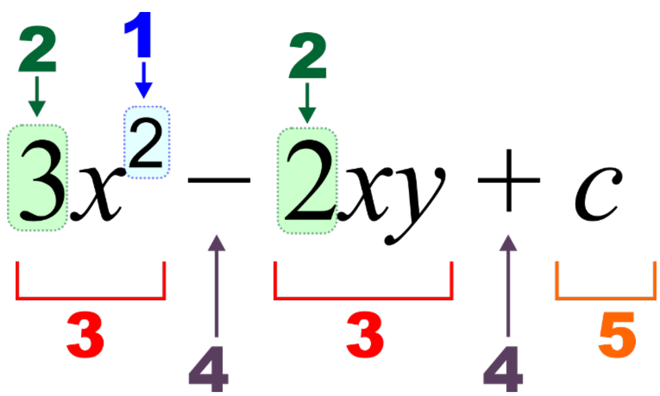
Algebra II



Day 1	Day 2	Day 3	Day 4	Day 5
Unit 4 **Topic 7** **Lesson 8** Attributes and Transformations of the natural Logarithmic Function Translations	**Unit 4** **Topic 7** **Lesson 6** Properties of Logarithms Simplify & Expand	**Unit 4** **Topic 7** **Lesson 6** Properties of Logarithms Simplify & Expand	**Unit 4** **Topic 7** **Lesson 9** Exponential and Logarithmic Equations	**Unit 4** **Topic 7** **Lesson 9** Exponential and Logarithmic Equations
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
Unit 4 **Topic 7** **Lesson 9** Exponential and Logarithmic Equations Applications	**Unit 4** **Topic 7** **Lesson 9** Exponential and Logarithmic Equations	**Unit 5** **Topic 8** **Lesson 1** Attributes of Polynomial Functions	**Unit 5** **Topic 8** **Lesson 1** Attributes of Polynomial Functions	**Unit 5** **Topic 8** **Lesson 2** Add, Subtract, and Multiply Polynomials

Name ______________________________ Class ____________ Date ____________

7-6 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m + \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$ — Use the Power Property twice. $6^2 = 36,\ 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$= \log_2 36 - \log_2 9 + \log_2 3$

$= (\log_2 36 - \log_2 9) + \log_2 3$ — Group two of the logarithms. Use order of operations.

$= \log_2 \frac{36}{9} - \log_2 3$ — Quotient Property

$= \log_2 \left(\frac{36}{9} \cdot 3\right)$ — Product Property

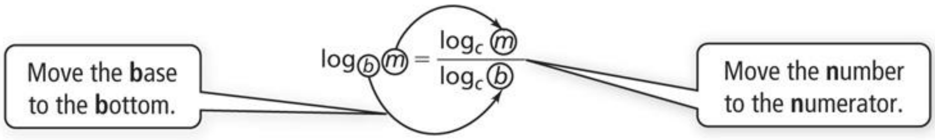
$= \log_2 12$ — Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

7-6 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \frac{\log_2 8}{\log_2 4}$ The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$= \frac{3}{2}$ Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ **2.** $2 \log x + \log 5$ **3.** $\log_4 2 - \log_4 6$

4. $3 \log_3 3 - \log_3 3$ **5.** $\log_5 8 + \log_5 x$ **6.** $\log 2 - 2 \log x$

7. $\log_2 x + \log_2 y$ **8.** $3 \log_7 x - 5 \log_7 y$ **9.** $4 \log x + 3 \log x$

10. $\log_5 x + 3 \log_5 y$ **11.** $3 \log_2 x - \log_2 y$ **12.** $\log_2 16 - \log_2 8$

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint:* Common logarithms are logarithms with base 10.)

13. $\log_4 12$ **14.** $\log_2 1000$ **15.** $\log_5 16$

16. $\log_{11} 205$ **17.** $\log_9 32$ **18.** $\log_{100} 51$

Name ____ Class _______________ Date _______________

7-8 Reteaching

Attributes and Transformations of the Natural Logarithm Function

Natural logarithmic functions are the inverse of $a = e^b$. This equation can be written as $b = \log_e a$ and also as $b = \ln a$. The term "ln" means natural logarithm.

The natural logarithm function has the domain $(0, \infty)$, the range of all real numbers, one or no x-intercept, and an asymptote of $x = 0$.

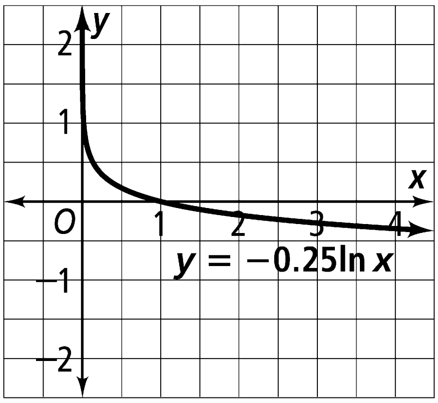
Problem

Graph the function $y = -0.25 \ln x$ and analyze the domain, range, intercepts, and asymptote.

The first step in graphing the function is to make a table of values. Use values for x that work well in the function.

x	$y = -0.25\ln x$
–1	undefined
0	undefined
$\frac{1}{e}$	0.25
e	0
e^2	–0.5

Next, sketch a graph of the function. Plot each point and connect the points.



From the graph, you can determine that the domain is $(0, \infty)$, the range is all real numbers, the x-intercept is $(1, 0)$, and the asymptote is $x = 0$.

Exercises

Find the domain, range, x-intercept, and asymptote for each function.

1. $y = \ln x + 3$

2. $y = 4 \log_e x$

3. $y = -0.6 \ln x$

4. $y = -3 \log_e x + 4$

7-8 Reteaching (continued)

Attributes and Transformations of the Natural Logarithm Function

Just as in other functions, transformations can be used to map the parent function to the graph of a natural logarithm function.

Natural logarithms have the same set of transformations as other logarithm bases.

$f(x) = a \log_e x$	for absolute $a > 1$, graph shows vertical stretch for $0 <$ absolute $a < 1$, graph shows vertical compression or shrink for $a < 0$, graph shows reflection across the x-axis
$f(x) = \log_e x + d$	for $d > 0$, graph shifts up d units for $d < 0$, graph shifts down d units

Problem

The time T in years at which an investment is worth x dollars is given by the function $T = (-2 \log_e x) + 10$. Describe the graph of this function as a translation, stretch, compression, or reflection of the parent function, $T = \log_e x$.

Compare the parent function $T = \log_e x$ to the transformed function using the transformation of a log function, $f(x) = a \log_e x + d$.

In this case $f(x)$ is replaced by T, so $T = a \log_e + d$.

The value of a in $T = (-2 \log_e x) + 10$ is -2. The absolute value of -2 is greater than 1, so the graph shows a vertical stretch. Because $a < 0$, the graph is also reflected across the x-axis.

The value of d in $T = (-2 \log_e x) + 10$ is 10. Because $d > 0$, the graph shows a translation of 10 units up.

Exercises

Identify the transformation that maps the first function to the second function.

5. $f(x) = \log_e x,\ g(x) = \frac{1}{3}\log_e x - 3$

6. $f(x) = \ln x$, $g(x) = -4 \ln x + 16$

7. $f(x) = \log_e x$, $g(x) = -(\log_e x) + 1$

8. $f(x) = 3 \ln x$, $g(x) = 6 \ln x - 2$

Name ______________________________________ Class _______________ Date _______________

7-9 Reteaching

Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem

What is the solution of $7 - 5^{2x-1} = 4$?

$7 - 5^{2x-1} = 4$	
$-5^{2x-1} = -3$	First isolate the term that has the variable in the exponent. Begin by subtracting 7 from each side.
$5^{2x-1} = 3$	Multiply each side by –1.
$\log_5 5^{2x-1} = \log_5 3$	Because the variable is in the exponent, use logarithms. Take $\log_5$ of each side because 5 is the base of the exponent.
$(2x-1)\log_5 5 = \log_5 3$	Use the Power Property of Logarithms.
$2x - 1 = \log_5 3$	Simplify. (Recall that $\log_b b = 1$.)
$2x - 1 = \dfrac{\log 3}{\log 5}$	Apply the Change of Base Formula.
$2x = \dfrac{\log 3}{\log 5} + 1$	Add 1 to each side.
$x = \dfrac{1}{2}\left(\dfrac{\log 3}{\log 5} + 1\right)$	Divide each side by 2.
$x \approx 0.84$	Use a calculator to find a decimal approximation.

Exercises

Solve each equation. Round the answer to the nearest hundredth.

1. $2^x = 5$

2. $10^{2x} = 8$

3. $5^{x+1} = 25$

4. $2^{x+3} = 9$

5. $3^{2x-3} = 7$

6. $4^x - 5 = 3$

7. $5 + 2^{x+6} = 9$

8. $4^{3x} + 2 = 3$

9. $1 - 3^{2x} = -5$

10. $2^{3x} - 2 = 13$

11. $5^{2x+7} - 1 = 8$

12. $7 - 2^{x+7} = 5$

Name ____ Class _______________ Date _______________

7-9 Reteaching (continued)

Exponential and Logarithmic Equations

Use exponents to solve logarithmic equations.

Problem

What is the solution of $8 - \log(4x - 3) = 4$?

$8 - 2\log(4x - 3) = 4$	
$-\log(4x - 3) = -4$	First isolate the term that has the variable in the logarithm. Begin by subtracting 8 from each side.
$\log(4x - 3) = 4$	Multiply each side by –1.
$4x - 3 = 10^4$	Write in exponential form.
$4x - 3 = 10{,}000$	Simplify.
$4x = 10{,}003$	Add 3 to each side.
$x = \frac{10{,}003}{4}$	Solve for x.
$x = 2500.75$	Divide.

Exercises

Solve each equation. Round the answer to the nearest thousandth.

13. $\log x = 2$

14. $\log 3x = 3$

15. $\log 2x + 2 = 6$

16. $5 + \log(2x + 1) = 6$

17. $\log 5x + 62 = 62$

18. $6 - \log\frac{1}{2}x = 3$

19. $\log(4x - 3) + 6 = 4$

20. $\frac{2}{3}\log 5x = 2$

21. $2\log 250x - 6 = 4$

22. $5 - 2\log x = \frac{1}{2}$

Name ______________________________ Class ____________ Date ____________

7-10 Reteaching

Natural Logarithms

The **natural logarithmic function** is a logarithm with base e, an irrational number.

You can write the natural logarithmic function as $y = \log_e x$, but you usually write it as $y = \ln x$.

$y = e^x$ and $y = \ln x$ are inverses, so if $y = e^x$, then $x = \ln y$.

To solve a natural logarithm equation:

- If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.
- If term containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem

What is the solution of $4e^{2x} - 2 = 3$?

Step 1 Isolate the term containing the variable on one side of the equation.

$$4e^{2x} - 2 = 3$$

$$4e^{2x} = 5$$ Add 2 to each side of the equation.

$$e^{2x} = \frac{5}{4}$$ Divide each side of the equation by 4.

Step 2 Take the natural logarithm of each side of the equation.

$$\ln(e^{2x}) = \ln\left(\frac{5}{4}\right)$$

$$2x = \ln\left(\frac{5}{4}\right)$$ Definition of natural logarithm

Step 3 Solve for the variable.

$$x = \frac{\ln\left(\frac{5}{4}\right)}{2}$$ Divide each side of the equation by 2.

$$x \approx 0.112$$ Use a calculator.

Step 4 Check the solution.

$$4e^{2(0.112)} - 2 \stackrel{?}{=} 3$$

$$4e^{0.224} - 2 \stackrel{?}{=} 3$$

$$3.004 \approx 3$$

The solution is $x \approx 0.112$.

Name ______________________________ Class ______________ Date ______________

7-10 Reteaching (continued)

Natural Logarithms

Problem

What is the solution of $\ln(t - 2)^2 + 1 = 6$? Round your answer to the nearest thousandth.

Step 1 Isolate the term containing the variable on one side of the equation.

$$\ln(t - 2)^2 + 1 = 6$$

$\ln(t - 2)^2 = 5$ Subtract 1 from each side of the equation.

Step 2 Raise each side of the equation to the base *e*.

$e^{\ln(t-2)^2} = e^5$ Definition of natural logarithm

$(t - 2)^2 = e^5$

Step 3 Solve for the variable.

$t - 2 = \pm e^{\frac{5}{2}}$ Take the square root of each side of the equation.

$t = 2 \pm e^{\frac{5}{2}}$ Add 2 to each side of the equation.

$t \approx 14.182 \text{ or } -10.182$ Use a calculator.

Step 4 Check the solution.

$\text{In}\ (14.182 - 2)^2 \quad 5$ $\text{In}\ (-10.182 - 2)^2 \quad 5$

$4.9999 \approx 5$ $4.9999 \approx 5$

The solutions are $t \approx 14.182$ and -10.182.

Exercises

Use natural logarithms to solve each equation. Round your answer to the nearest thousandth. Check your answers.

1. $2e^x = 4$ **2.** $e^{4x} = 25$ **3.** $e^x = 72$

4. $e^{3x} = 124$ **5.** $12e^{3x-2} = 8$ **6.** $\frac{1}{2}e^{6x} = 5$

Solve each equation. Round your answer to the nearest thousandth. Check your answers.

7. $\ln(x - 3) = 2$ **8.** $\ln 2t = 4$ **9.** $1 + \ln x^2 = 2$

10. $\ln(2x - 5) = 3$ **11.** $\frac{1}{3}\ln 2t = 1$ **12.** $\ln(t - 4)^2 + 2 = 5$

Name ___________________________ Class ___________ Date ___________

8-1 Reteaching

Polynomial Functions

Problem

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^6$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

Step 3 Count the number of terms in the simplified polynomial. It has 5 terms.

Step 4 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (n) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
a (+3) is positive and n (6) is even, so the end behavior is up and up.

Exercises

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$ **2.** $15x^7 - 7$ **3.** $2x - 6x^2 - 9$

8-1 Reteaching (continued)

Polynomial Functions

Problem

x	y
–3	52(y_1)
–2	18(y_2)
–1	2(y_3)
0	–2 (y_4)
1	0 (y_5)
2	2 (y_6)
3	–2 (y_7)

What is the degree of the polynomial function that generates the data shown at the right? What are the differences when they are constant? To find the degree of a polynomial function from a data table, you can use the differences of the *y*-values.

Step 1 Determine the values of $y_2 - y_1$, $y_3 - y_2$, $y_4 - y_3$, $y_5 - y_4$, $y_6 - y_5$, $y_7 - y_6$. These are called the first differences. Make a new column using these values.

Step 2 Continue determining differences until the *y*-values are all equal. The quantity of differences is the degree of the polynomial function. The third differences are all equal so this is a third degree polynomial function. The value of the third differences is –6.

x	y	1st diff.
−3	52 (y_1)	
−2	18 (y_2)	−34 (y_8)
−1	2 (y_3)	−16 (y_9)
0	−2 (y_4)	−4 (y_{10})
1	0 (y_5)	2 (y_{11})
2	2 (y_6)	2 (y_{12})
3	−2 (y_7)	−4 (y_{13})

x	y	1st diff.	2nd diff.	3rd diff.
−3	52 (y_1)	−34 (y_8)		
−2	18 (y_2)	−16 (y_9)	18	
−1	2 (y_3)	−4 (y_{10})	12	−6
0	−2 (y_4)	2 (y_{11})	6	−6
1	0 (y_5)	2 (y_{12})	0	−6
2	2 (y_6)	−4 (y_{13})	−6	−6
3	−2 (y_7)			

Exercises

What is the degree of the polynomial function that generates the data in the table? What are the differences when they are constant?

4.

x	y
–3	216
–2	24
–1	0
0	0
1	0
2	–24
3	–216

5.

x	y
–3	–101
–2	–37
–1	–11
0	–5
1	–1
2	19
3	73

6.

x	y
–3	6
–2	26
–1	8
0	0
1	2
2	–34
3	–204

Name ____ Class ______________ Date _______________

8-2 Reteaching

Adding, Subtracting, and Multiplying Polynomials

You can multiply a monomial and a trinomial by solving simpler problems. You can use the Distributive Property to make three simpler multiplication problems.

Problem

What is the simplified form of $3x(2x^2 + 4x - 1)$?

Use the Distributive Property to rewrite the problem as three separate multiplication problems.

$$3x(2x^2 + 4x - 1) = (3x \cdot 2x^2) + (3x \cdot 4x) + (3x \cdot (-1))$$

Remember that when you multiply same-base terms containing exponents, you add the exponents.

Solve	$3x \cdot 2x^2 = 6x^3$	Multiply inside the first pair of parentheses.
	$3x \cdot 4x = 12x^2$	Multiply inside the second pair of parentheses.
	$3x \cdot (-1) = -3x$	Multiply inside the third pair of parentheses.
	$6x^3 + 12x^2 - 3x$	Add the products.
Check	$6x^3 \div 2x^2 = 3x$	Check your solution using division.
	$12x^2 \div 4x = 3x$	
	$-3x \div (-1) = 3x$	

Solution: $3x(2x^2 + 4x - 1) = 6x^3 + 12x^2 - 3x$

Exercises

Simplify each product.

1. $4x(2x - 7)$

2. $3y(3y + 4)$

3. $2z^2(2z - 3)$

4. $3a(-4a - 6)$

5. $6b(3b^2 + 2b - 4)$

6. $3c^2(2c^2 - 4c + 3)$

7. $-2d(4d^2 + 3d - 2)$

8. $5e^2(-3e^2 - 2e - 3)$

9. $4f(-3f^3 + 2f^2 + 6)$

Name ____ Class _______________ Date _______________

8-2 Reteaching (continued)

Adding, Subtracting, and Multiplying Polynomials

To subtract polynomials, follow the same steps as in addition.

Problem

What is the simplified form of $(6x^3 + 4x^2 - 3x) - (2x^3 + 3x^2 - 5x)$?

Write the problem vertically, lining up the like terms.
Then subtract each pair of like terms.

$$\begin{array}{r} 6x^3 + 4x^2 - 3x \\ -(2x^3 + 3x^2 - 5x) \\ \hline \end{array}$$

Solve

Subtract the x^3 terms.
$6x^3 - 2x^3 = 4x^3$

Subtract the x^2 terms.
$4x^2 - 3x^2 = x^2$

Subtract the x terms.
$-3x - (-5x) = 2x$

$$\begin{array}{r} 6x^3 + 4x^2 - 3x \\ -(2x^3 + 3x^2 - 5x) \\ \hline 4x^3 + x^2 + 2x \end{array}$$

Add the differences.

Check Check your solution using subtraction.

$4x^3 + 2x^3 = 6x^3$ $x^2 + 3x^2 = 4x^2$ $2x + (-5x) = -3x$

Solution: $(6x^3 + 4x^2 - 3x) - (2x^3 + 3x^2 - 5x) = 4x^3 + x^2 + 2x$

Exercises

Simplify.

10. $$\begin{array}{r} 4k^2 + 5k \\ -(3k^2 + 2k) \\ \hline \end{array}$$

11. $$\begin{array}{r} 5m^2 - 4m \\ -(2m^2 + 3m) \\ \hline \end{array}$$

12. $$\begin{array}{r} 7n^2 + 4n + 9 \\ -(4n^2 + 3n + 5) \\ \hline \end{array}$$

13. $$\begin{array}{r} 5p^2 + 6p + 4 \\ -(7p^2 + 4p + 8) \\ \hline \end{array}$$

14. $$\begin{array}{r} 3q^3 + 2q^2 + 7q \\ -(6q^3 - 4q^2 + 5q) \\ \hline \end{array}$$

15. $$\begin{array}{r} 2r^3 - 2r^2 + 5r \\ -(4r^3 + 5r^2 + 3r) \\ \hline \end{array}$$

16. $(6s^2 - 5s) - (-2s^2 + 3s)$

17. $(3w^2 + 6w - 5) - (5w^2 - 4w + 2)$